

General Certificate of Education  
January 2007  
Advanced Level Examination



**MATHEMATICS**  
**Unit Pure Core 4**

**MPC4**

Thursday 25 January 2007 9.00 am to 10.30 am

**For this paper you must have:**

- an 8-page answer book
  - the **blue** AQA booklet of formulae and statistical tables.
- You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

**Instructions**

- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MPC4.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.

**Information**

- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

**Advice**

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

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Answer **all** questions.

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1 A curve is defined by the parametric equations

$$x = 1 + 2t, \quad y = 1 - 4t^2$$

(a) (i) Find  $\frac{dx}{dt}$  and  $\frac{dy}{dt}$ . (2 marks)

(ii) Hence find  $\frac{dy}{dx}$  in terms of  $t$ . (2 marks)

(b) Find an equation of the normal to the curve at the point where  $t = 1$ . (4 marks)

(c) Find a cartesian equation of the curve. (3 marks)

2 The polynomial  $f(x)$  is defined by  $f(x) = 2x^3 - 7x^2 + 13$ .

(a) Use the Remainder Theorem to find the remainder when  $f(x)$  is divided by  $(2x - 3)$ . (2 marks)

(b) The polynomial  $g(x)$  is defined by  $g(x) = 2x^3 - 7x^2 + 13 + d$ , where  $d$  is a constant.

Given that  $(2x - 3)$  is a factor of  $g(x)$ , show that  $d = -4$ . (2 marks)

(c) Express  $g(x)$  in the form  $(2x - 3)(x^2 + ax + b)$ . (2 marks)

3 (a) Express  $\cos 2x$  in terms of  $\sin x$ . (1 mark)

(b) (i) Hence show that  $3 \sin x - \cos 2x = 2 \sin^2 x + 3 \sin x - 1$  for all values of  $x$ . (2 marks)

(ii) Solve the equation  $3 \sin x - \cos 2x = 1$  for  $0^\circ < x < 360^\circ$ . (4 marks)

(c) Use your answer from part (a) to find  $\int \sin^2 x \, dx$ . (2 marks)

- 4 (a) (i) Express  $\frac{3x-5}{x-3}$  in the form  $A + \frac{B}{x-3}$ , where  $A$  and  $B$  are integers. (2 marks)
- (ii) Hence find  $\int \frac{3x-5}{x-3} dx$ . (2 marks)
- (b) (i) Express  $\frac{6x-5}{4x^2-25}$  in the form  $\frac{P}{2x+5} + \frac{Q}{2x-5}$ , where  $P$  and  $Q$  are integers. (3 marks)
- (ii) Hence find  $\int \frac{6x-5}{4x^2-25} dx$ . (3 marks)
- 5 (a) Find the binomial expansion of  $(1+x)^{\frac{1}{3}}$  up to the term in  $x^2$ . (2 marks)
- (b) (i) Show that  $(8+3x)^{\frac{1}{3}} \approx 2 + \frac{1}{4}x - \frac{1}{32}x^2$  for small values of  $x$ . (3 marks)
- (ii) Hence show that  $\sqrt[3]{9} \approx \frac{599}{288}$ . (2 marks)
- 6 The points  $A$ ,  $B$  and  $C$  have coordinates  $(3, -2, 4)$ ,  $(5, 4, 0)$  and  $(11, 6, -4)$  respectively.
- (a) (i) Find the vector  $\overrightarrow{BA}$ . (2 marks)
- (ii) Show that the size of angle  $ABC$  is  $\cos^{-1}\left(-\frac{5}{7}\right)$ . (5 marks)
- (b) The line  $l$  has equation  $\mathbf{r} = \begin{bmatrix} 8 \\ -3 \\ 2 \end{bmatrix} + \lambda \begin{bmatrix} 1 \\ 3 \\ -2 \end{bmatrix}$ .
- (i) Verify that  $C$  lies on  $l$ . (2 marks)
- (ii) Show that  $AB$  is parallel to  $l$ . (1 mark)
- (c) The quadrilateral  $ABCD$  is a parallelogram. Find the coordinates of  $D$ . (3 marks)

**Turn over for the next question**

**Turn over ►**

- 7 (a) Use the identity

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

to express  $\tan 2x$  in terms of  $\tan x$ .

(2 marks)

- (b) Show that

$$2 - 2 \tan x - \frac{2 \tan x}{\tan 2x} = (1 - \tan x)^2$$

for all values of  $x$ ,  $\tan 2x \neq 0$ .

(4 marks)

- 8 (a) (i) Solve the differential equation  $\frac{dy}{dt} = y \sin t$  to obtain  $y$  in terms of  $t$ . (4 marks)

- (ii) Given that  $y = 50$  when  $t = \pi$ , show that  $y = 50e^{-(1+\cos t)}$ . (3 marks)

- (b) A wave machine at a leisure pool produces waves. The height of the water,  $y$  cm, above a fixed point at time  $t$  seconds is given by the differential equation

$$\frac{dy}{dt} = y \sin t$$

- (i) Given that this height is 50 cm after  $\pi$  seconds, find, to the nearest centimetre, the height of the water after 6 seconds. (2 marks)

- (ii) Find  $\frac{d^2y}{dt^2}$  and hence verify that the water reaches a maximum height after  $\pi$  seconds. (4 marks)

**END OF QUESTIONS**